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SOME APPROACHES TO SOLVE A COMPLEX PROBLEM OF GEOMETRICAL COVERING AND ORTHOGONAL CUTTING

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Abstract. Under consideration there are some approaches to solving a multi-criterion complex problem of geometrical covering the multilinked domain with orthogonal material succeed by cutting. The above problem is widely interpreted. To solve the problem the double-step approach is applied, that is, decomposition of the domain into rectangles not surpassing the domain dimension and the cutting itself. The approach suggested is compared to the three-step method described earlier.

Keywords: multi-criterion complex problem, complex problem of geometrical covering and cutting, packing, geometrical covering, covering orthogonal area

1. INTRODUCTION

It often happens that it becomes necessary to cover a certain flat area, probably with impediments, by some objects of smaller size [1]. The problem of geometrical covering is a special case of the NP-hard class of "cutting-packing problems" [2].

The paper considers the problem that takes place in the building industry. It is a complex problem of covering the preset geometrical area by some material and of cutting that material. It occurs, for example, when it is necessary to cover the floors with linoleum, fibreboard and some other materials. And at that we must minimize the material used while covering the area and maximize the sizes of the covering units.

For the first time the complex problem of geometrical covering was described in paper [3].
The three-step approach and the algorithms for solution are given in paper [4], they allow to estimate the partial information received in the process of solution.

While solving such NP-hard problems the issue of the day becomes the creation of new approach that can use the fast heuristic and meta-heuristic algorithms. We suggest in the paper that the existing approach to solving the complex problem should be improved.

2. A MULTI-CRITERIA COMPLEX PROBLEM OF GEOMETRICAL COVERING OF MULTI-CONNECTED DOMAIN BY ORTHOGONAL MATERIAL

Complex problem statement: there are one or several types of the orthogonal material for covering some orthogonal area. Find a plan of covering the area with rectangular items and a plan of cutting the material into covering items. At that the length of the covering items junctions and the material to be cut must be minimized.

Two interrelated sub-problems are to be considered while investigating the above problem, namely: the problem of geometrical covering and the one of orthogonal resource cutting. The covering area can be interpreted as a multiply-connected orthogonal polygon containing prohibited zones [5]. The orthogonal material is a roll (semi-endless strip) or a set of sheets of the given sizes.

Mathematical model of the complex problem of geometrical covering of multiply connected area with orthogonal material is stated as a double-criteria problem.

Given: a multiply connected covering area $P = \left\langle W, L, (\chi_\nu, \eta_\nu), (\omega_\nu, \lambda_\nu), \nu = \overline{1, \mu} \right\rangle$ of the width W and the length L, and also a set B of rectangular impediments of the given sizes $\omega_\nu, \lambda_\nu, \nu = \overline{1, \mu}$, here μ is a number of impediments. We enter the rectangular coordinate system: here the axes Ox and Oy coincide correspondingly with the bottom and the left flanks of the rectangle P that goes round the covering area. Location of each rectangular impediment Bv is defined by coordinates (χ_v, η_v) of its bottom left corner.

Besides there is an unlimited number of rectangular sheets of the length Θ and of the width h (for problems with the resource in the form of sheets), or there is the width h of the roll resource to be cut into rectangular items. There is also some information about waste material in the form of a set $K = (k_1, k_2, ..., k_\tau)$ of rectangular items with $kl_j \times kw_j, j = \overline{1,\tau}$ sizes, interpreted as some material of non-standard size.

Required: find a plan of covering of the multiply connected orthogonal area, a plan of resource cutting; minimize values of the functions that characterize the efficiency of solution for geometrical covering and cutting sub-problems:

$$\min F_{\text{cov}}(RT) = \sum_{i=1}^{m}(l_i + w_i),$$

$$\min F_{\text{cut}}(RT) = \begin{cases} N^{РЛ}, \text{ for rectangular sheets;} \\ L^{РП} = \max\limits_{i=1..m}(x_i' + l_i), \text{ for semi-endless strip;} \end{cases}$$

Here

$RT = \{m, w, l\}, w = (w_1, w_2, ..., w_m), l = (l_1, l_2, ..., l_m)$ is a set of m rectangular items of the sizes $w_i \times l_i, i = \overline{1,m}$, forming the covering of the given area, these items are to be cut out of the given orthogonal material. L_i, w_i represent the length and width of the covering rectangular item $rt_i \in RT$ included into the polygon covering, $RT = Q \cup K^+$, here Q is a set of covering P rectangular items having been cut out of the orthogonal resource, K^+ is a set of covering P rectangular items, cut out of the waste material.

Let us enter in some additional variables z_{ij}^x, z_{ij}^y, $\tilde{z}_{iv}^x$ and $\tilde{z}_{iv}^y$, so that $z_{ij}^x = 1$ ($z_{ij}^y = 1$), if the rectangular item i projection on the axis Ox (Oy) is on the left (lower) of the rectangular item j projection on the axis Ox (Oy), otherwise $z_{ij}^x = 0$ ($z_{ij}^y = 0$); $\tilde{z}_{iv}^x = 1$ ($\tilde{z}_{iv}^y = 1$), if the rectangular item i projection on the axis Ox (Oy) is on the left (lower) of the impediment v projection on the axis Ox (Oy), otherwise $\tilde{z}_{iv}^x = 0$ ($\tilde{z}_{iv}^y = 0$).

It is necessary to find the following:

1. A set RT of rectangular items that cover the P area and the corresponding covering plan, i.e. for every $rt_i \in RT$ we are to find a set of values $\langle x_i, y_i, l_i, w_i \rangle$, here x_i, y_i are coordinates of the item's left bottom corner in the coordinate system of the area P; l_i and w_i are correspondingly the length and width of a rectangular item so that:

$$l_i \le h,\ w_i \le \Theta,\ i = \overline{1,m}, \quad (1)$$

$$x_i + l_i \le L,\ y_i + w_i \le W,\ i = \overline{1,m}, \quad (2)$$

$$x_i \ge 0,\ y_i \ge 0,\ i = \overline{1,m}, \quad (3)$$

$$z_{ij}^x + z_{ji}^x + z_{ij}^y + z_{ji}^y \ge 1,\ i = \overline{1,m}, j = \overline{1,m}, i \ne j, \quad (4)$$

$$y_j \ge y_i + w_i - W(1 - z_{ij}^y),\ i = \overline{1,m}, j = \overline{1,m}, \quad (5)$$

$$x_j \ge x_i + l_i - L(1 - z_{ij}^x),\ i = \overline{1,m}, j = \overline{1,m}, \quad (6)$$

$$\tilde{z}_{iv}^x + \tilde{z}_{vi}^x + \tilde{z}_{iv}^y + \tilde{z}_{vi}^y \ge 1,\ i = \overline{1,m}, v = \overline{1,\mu}, \quad (7)$$

$$\eta_v \ge y_i + w_i - W(1 - \tilde{z}_{iv}^y),\ i = \overline{1,m}, v = \overline{1,\mu}, \quad (8)$$

$$\chi_v \ge x_i + l_i - L(1 - \tilde{z}_{iv}^x),\ i = \overline{1,m}, v = \overline{1,\mu}, \quad (9)$$

$$\sum_{i=1}^{m} w_i \cdot l_i = S_{МОП}. \quad (10)$$

The conditions (1) impose restrictions on the sizes of covering rectangular items. The conditions (2) and (3) set an admissible position of the rectangular item in the covering domain. The conditions (4)–(6) provide non-overlapping of rectangular items among themselves, the conditions (7)–(9) – provide non-overlapping of rectangular items with impediments. The condition (10) describes necessity of full geometrical covering of the useful area with rectangular items.

1. The plan to cut the orthogonal resource into rectangular items of the set RT is such, that the following conditions are satisfied:

$$x_i' \ge 0,\ y_i' \ge 0,\ i = \overline{1,m}, \quad (11)$$

$$x'_i + l_i \le \Theta,\ y'_i + w_i \le h,\ i = \overline{1,m}, \quad (12)$$

$$z_{ij}^x + z_{ji}^x + z_{ij}^y + z_{ji}^y \ge 1,\ i = \overline{1,m}, j = \overline{1,m}, i \ne j, \quad 13)$$

$$y_j' \ge y_i' + w_i - W(1 - z_{ij}^y),\ i = \overline{1,m}, j = \overline{1,m}, \quad (14)$$

$$x_j' \ge x_i' + l_i - L(1 - z_{ij}^x),\ i = \overline{1,m}, j = \overline{1,m}. \quad (15)$$

The conditions (11)–(12) provide admissible accommodation of rectangular items inside of the orthogonal material; the conditions (13)–(15) provide non-overlapping of rectangular items among themselves.

3. APPROACHES TO SOLVE A COMPLEX PROBLEM OF GEOMETRICAL COVERING AND CUTTING

The complex problem of geometrical covering and cutting represents the consecutive solution of several sub-problems. Splitting of an initial problem into sub-problems can be carried out in several ways.

Thus, in the paper [3] it is offered the three-step approach to solution of a complex problem of geometrical covering and cutting, namely: decomposition of multiply connected orthogonal covering area into a minimum number of rectangular boxes; definition of the covering plan o and the sizes of covering items; definition of the cutting plan of rectangular items out of some orthogonal material (Fig. 1).

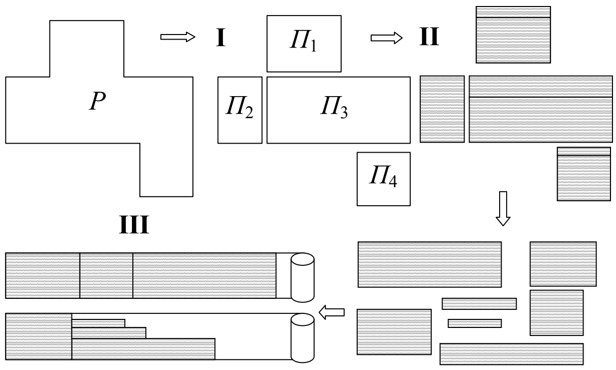

Fig. 1. Three steps of a complex problem solving: I is decomposition of a polygon into boxes; II is definition of covering plan; III is definition of cutting plan

The matrix decomposition method [4] was earlier suggested to solve the problems of decomposition and allocation of rectangles in multiply connected polygon. It can be applied as a procedure to solve some decomposition sub-problems when solving the complex problem. Based on the above method the algorithm with waste evaluation and the hybrid evolutional algorithm with values [4, 6] have been developed and successfully applied. In both algorithms the complex problem is solved in three steps.

However, one should think about the efficiency of such a three-step approach, because the resource parameters while decomposing the covering area into boxes are not taken into consideration. To increase the solution efficiency it is offered to unite stages of decomposing and covering plan defining, i. e. to decompose into boxes the sizes of which do not exceed the sizes of the orthogonal material. For this purpose it is offered to use the method of matrix decomposition [5] with modifications.

We suggest a new approach for searching the rational solution of a complex problem in two steps. On the first step one should solve the problem of decomposing the area into rectangular boxes the sizes of which do not exceed the sizes of the given material. On the second step one should cut the given material into covering items the sizes of which are equal to the sizes of boxes.

4. DECOMPOSING THE DOMAIN INTO RECTANGULAR BOXES

Given is the multiply connected orthogonal area for covering

$$P = \left\langle W, L, (\chi_\nu, \eta_\nu), (\omega_\nu, \lambda_\nu), \nu = \overline{1, \mu} \right\rangle .$$

It is required to find the set $\Pi = \{\Pi_1, \Pi_2, \ldots, \Pi_k\}$ of rectangular boxes $\Pi_i = \left\langle (x^i, y^i), (W^i, L^i) \right\rangle$ of minimal capacity, here (x_i, y_i) are the coordinates of the bottom left corner of the i-th box, W^i and L^i are its width and length correspondingly, k is the capacity of a set Π, satisfying the following conditions:

- admissibility of the rectangular box location in the area of covering;
- non-overlapping of rectangular boxes among themselves;
- non-overlapping of rectangular boxes with impediments;
- full geometrical covering of the covering area to be used with rectangular boxes;
- $W^i \le h, L^i \le \Theta,\ i = \overline{1, k}$ – is an additional restriction; when it is fulfilled any rectangular item for covering can be entirely cut out of the orthogonal material.

Matrix decomposition method modification

Representation of the domain with impediments in the form of matrix. Let us draw lines through horizontal edges of impediments. As a result the domain with impediments is divided into G rectangles with the lengths L and the widths d_γ, $\gamma = \overline{1, G}$. If the unequality $d_\gamma > h$ is true for any rectangle, the latter is divided with a through horizontal line into two rectangles with the widths h and $(d_\gamma - h)$. Such horizontal dividing is produced till the condition $d_\gamma \le h, \gamma = \overline{1, G}$ is fulfilled.

We also draw lines through vertical edges of impediments. As a result the domain with impediments is divided into U rectangles with the lengths q_φ, $\varphi = \overline{1, U}$ and the widths W.

If a semi-endless strip serves as the orthogonal resource then the length of areas is not restricted. Otherwise, by analogy with the horizontal splitting, the vertical division proceeds until the condition $q_\varphi \le \Theta, \varphi = \overline{1, U}$ be satisfied.

As a result the domain with impediments is covered with a network each cell of which either does not contain any impediment (is empty), or represents an impediment or a part of it. Thus the sizes of cells do not exceed the sizes of the resource.

Let us attach number 0 to each cell (γ, σ) if it is free (empty), and number1 if it is an impediment. Then the initial area with impediments can be written down in the form of a matrix with Boolean variables.

1. Choice of initial cell. This step remains unchanged: we choose the left bottom cell of the initial area as an initial cell. If the cell is an impediment we move up the column till an empty cell is found and it is accepted as the initial one.

2. Aggregation of empty cells, starting with the initial one, by vertical, horizontal or diagonal way. The aggregation continues till one of the following condition is fulfilled:

- the upper bound of the covering area is reached;
- there is an impediment on the way of aggregation;
- the width of a box equals h;
- the length of a box equals Θ (for sheet resource).

3. "Level" detachment of empty rectangular domains. This step is also unchanged: when the upper bound of the domain with impediments is reached an imagined cut is done through the right bound of the longest domain. The detachment of boxes out of the domain goes on to the right of the cut. The matrix decomposition method (we come back to the step 2) is recursively applied the secondary covering areas received in the process. The algorithm works till there are no empty cells.

When the width of all boxes equals h, and the length equals Θ, the problem is considered to be solved, it means that every box can be covered without waste with a rectangular sheet of the resource. If the resource represents a semi-endless strip and the width of boxes equals h, the material for covering each box is achieved by one vertical cut of the roll resource

If there are some boxes with the sizes less than those of the resource, we go to the cutting step.

CONCLUSION

The offered approach permits to solve a complex problem of geometrical covering and cutting in two steps: definition of the sizes of covering elements and a resource cutting. It permits to reduce laboriousness of calculations. Owing to updating of the matrix decomposition method, the sizes of covering elements do not exceed the sizes of an orthogonal resource, and as the result of this subproblem solution we get a covering plan.

At the resource cutting step it is possible to apply any method most suitable for the certain applied situation. For example, the paper [7] describes the algorithm of a problem solving where the level-by-level algorithm with estimations is used at the cutting step.

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