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**INFLUENCE OF RAYLEIGH NUMBER ON THE PASSIVE COOLING OF A HEAT-GENERATING SOLID BODY****© M. Hussain<sup>1</sup>, T. I. Korchagin<sup>2</sup>, I. V. Miroshnichenko<sup>2\*</sup>,  
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*A numerical study has been conducted to assess the convective heat transfer inside an enclosure containing a heat-producing element. The vertical sides of an enclosure are kept at a constant temperature, while the horizontal boundaries are thermally insulated. Heat generating element is assumed to be placed in the central part of the considered cavity. The governing system for the considered problem is tackled by adopting the control volume approach. Rayleigh number ( $10^4 < Ra < 10^6$ ) and the walls emissivity ( $0 < \varepsilon < 1$ ) are the governing parameters. The influence of the specified parameters on the stream function and temperature isolines, as well as the convective and radiative Nusselt numbers, are presented. Results concluded that the surface emissivity of boundaries and Rayleigh number have a considerable impact on the cooling efficiency of the heat-producing body.*

**Keywords:** heat-generating element, natural convection, thermal radiation, finite volume method.

**Introduction**

Convective thermal transmission has been studied by several researchers in recent years because of its significant role in thermal transmission in a wide variety of scientific and manufacturing applications [1–5]. Managing heat in electronic components housed within sealed cabinets is a key challenge for designers, especially as miniaturization and increased operating temperatures push the limits of thermal control. In modeling, these cabinets are represented as rectangular spaces, with the electronic components modeled as obstacles, either as discrete bodies or partitions. These components may have heat-generating, constant heat flux, or isothermal thermal properties. Natural convection remains the preferred cooling method due to its affordability, silent operation, reliability, and straightforward design. However, the interaction between heat radiation and thermogravitational convection plays a critical role in energy transfer, particularly when surface emissivity is high. Therefore, radiation significantly affects overall heat transfer and must be taken into account [6–10].

Miroshnichenko and Sheremet [11] have presented a detailed review about theoretical and experimental works allied with thermal transmission natural convection in rectangular cavities. Various enclosure patterns have been examined, including varying initials and boundary conditions, heat source placements, and radiative features. Oh and colleagues [12] conducted a computational analysis to investigate the steady-state transmission of thermal energy and flow aspects of free convection within a vertical square cavity with a sustained temperature differential and a heat-producing conducting body within. In this study, the Ra is within the range  $10^3$  to  $10^4$ , whereas the ratio of the temperature difference ranges from 0 to 50. The Pr, aspect ratio, and ratio of conductivity remain 0.71, 0.25, and 1, respectively. They studied how the patterns of fluid flow and temperature distribution, as well as the average Nusselt numbers on the hot and cold surfaces, changed in relation to the ratio of temperature differences for different values of the Rayleigh number. Deng and colleagues [13] studied the interactions of local heat sources in enclosures. They used unified study of heat transport properties and a combined temperature scale technique for heat sources of different types, sizes, and intensities. For evaluating the interactions between several distinct heaters and their effects on the total patterns of thermal and fluid flow, this method is demonstrated to be both generic and efficient. In the convection process, the heat source located at the bottom significantly influences the flow of fluid. As a result, we observe a pattern consisting mainly of two main cells circulating around the internal volumetric heat sources. As a result, any imbalance between these internal heat sources has a considerable influence on the structure of flow in the conduction regime while having minimal impact in the convection zone.

Boukendil and colleagues [14] conducted a computational study to examine the laminar natural convective heat transfer and surface radiation around two square cylinders with different temperatures. They positioned the cylinders in a square cavity with perfectly insulating walls and analyzed how the governing parameters affected the flow patterns, temperature distribution, convective Nusselt numbers, and radiative Nusselt numbers. Their findings showed that surface radiation significantly influenced the flow structure inside the cavity, particularly at higher Rayleigh numbers. Additionally, the placement, aspect ratio, and diameter of the inner square cylinders had a substantial impact on both radiative and convective Nusselt numbers. Das and Reddy [15] modeled the natural convection flow in a square enclosure with a centrally located conducting square

block, adjusting both components at various inclination angles. The calculations were carried out using the finite volume method, which included a staggered grid arrangement and the SIMPLE algorithm. It was shown that for low Rayleigh numbers, the inclination angle has little effect on heat transfer for different values of thermal conductivity.

The current study emphasizes the consequences of cavity size and emissivity of surfaces on the heat transport within a square enclosure comprising a heat-generating element.

### Mathematical formulation

In the presence of a source of volumetric heat generation, the convective thermal transport problem with the considerations of thermal radiations in a closed cooled cavity is considered. The research focuses towards a 2D enclosure having dimension  $L \times L$  and heat generating element of size  $h \times h$  located in the middle of the considered cavity (Fig. 1). It is assumed that the rate at which heat is produced by the heat-producing element is uniform, whereas the vertical walls ( $x = 0, x = L$ ) are isothermal, and the horizontal walls of the enclosure ( $y = 0, y = L$ ) are adiabatic.

Air is employed as the participating fluid, with its thermophysical parameters considered constant except for density, which fluctuates with temperature using the Boussinesq approximation. The medium is deemed non-absorbing and non-emitting, making it radiation-inactive.

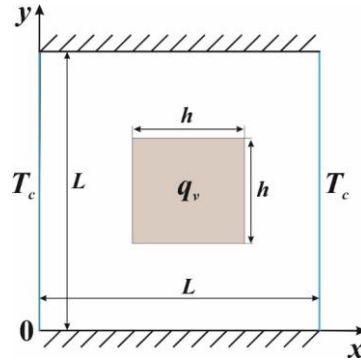


Figure 1. Domain of interest.

The conservation principles governing mass, momentum, and energy can be expressed through a system of differential equations using primitive variables:

continuity equation inside the air cavity

$$\frac{\partial u_1}{\partial x} + \frac{\partial u_2}{\partial y} = 0; \quad (1)$$

momentum equations inside the air cavity

$$\frac{\partial u_1}{\partial t} + u_1 \frac{\partial u_1}{\partial x} + u_2 \frac{\partial u_1}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \left( \frac{\partial^2 u_1}{\partial x^2} + \frac{\partial^2 u_1}{\partial y^2} \right); \quad (2)$$

$$\frac{\partial u_2}{\partial t} + u_1 \frac{\partial u_2}{\partial x} + u_2 \frac{\partial u_2}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial y} + \nu \left( \frac{\partial^2 u_2}{\partial x^2} + \frac{\partial^2 u_2}{\partial y^2} \right) + \beta(T - T_c) g; \quad (3)$$

energy equation inside the air cavity

$$\frac{\partial T}{\partial t} + u_1 \frac{\partial T}{\partial x} + u_2 \frac{\partial T}{\partial y} = a_f \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right); \quad (4)$$

heat conduction equation inside the heat source

$$\frac{\partial T}{\partial t} = a_s \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) + \frac{q_v}{\rho c_p}. \quad (5)$$

The presented system of equations considers a two-dimensional conjugate heat transfer problem with two physically distinct regions: the air cavity and the solid heat source. The system contains five independent equations: the continuity equation, two momentum conservation equations, the energy equation for the air in the cavity, and the heat conduction equation for the solid.

It is necessary to add the conditions of uniqueness, namely the boundary and initial conditions to the above-presented defining equations:

$$\begin{aligned}
 t = 0: u_1 = u_2 = 0, & \quad T = T_c, & \quad \text{at } 0 \leq x \leq L; \quad 0 \leq y \leq L \\
 t > 0: u_1 = u_2 = 0, & \quad T = T_c, & \quad \text{at } x = 0, L; \quad 0 \leq y \leq L \\
 u_1 = u_2 = 0, & \quad -\lambda_f \frac{\partial T}{\partial y} + q_{rad} = 0, & \quad \text{at } y = 0; \quad 0 \leq x \leq L \\
 u_1 = u_2 = 0, & \quad \lambda_f \frac{\partial T}{\partial y} + q_{rad} = 0, & \quad \text{at } y = L; \quad 0 \leq x \leq L \\
 u_1 = u_2 = 0, & \quad \lambda_f \frac{\partial T}{\partial n} + q_{rad} = \lambda_s \frac{\partial T}{\partial n}, & \quad \text{at air/solid interfaces}
 \end{aligned} \tag{6}$$

Here  $x, y$  – dimensional Cartesian coordinates,  $u_1, u_2$  – velocity components along  $x$  and  $y$  axis,  $t$  – time,  $T$  – temperature,  $g$  – acceleration of gravity,  $p$  – pressure,  $\beta$  – coefficient of volumetric thermal expansion,  $\rho$  – density,  $a_f$  – air thermal diffusivity,  $a_s$  – solid thermal diffusivity,  $c_p$  – specific heat,  $\nu$  – kinematic viscosity,  $q_v$  – volume density of heat flux,  $\lambda_s$  – solid thermal conductivity,  $\lambda_f$  – air thermal conductivity.

Thermal radiation plays a crucial role in heat transfer within enclosures. Several factors influence the degree of radiative heat transfer, such as the geometry of the enclosure, wall temperatures, surface emissivity, and the thermophysical properties of the fluid (e.g., air). In investigations regarding free convection, the radiation is sometimes disregarded due to the high computational resources required for its analysis. However, practical observations demonstrate that radiative thermal transport has a considerable impact on the system and should not be overlooked. For good reproducibility of the results, the values of the key parameters of the problem are presented in *Table 1*.

Table 1

Values of key physical parameters of the problem under consideration

Parameter	Air	Solid
$\nu, \text{m}^2/\text{s}$	$1,51 \cdot 10^{-5}$	-
$\beta, \text{K}^{-1}$	$3,67 \cdot 10^{-3}$	-
$c_p, \text{Дж}/(\text{кг} \cdot \text{°C})$	1 005 Дж/(кг·°C)	700
$\lambda_s, \text{Вт}/(\text{м} \cdot \text{K})$	-	150
$\lambda_f, \text{Вт}/(\text{м} \cdot \text{K})$	0,026	-

The formulated problem is solved numerically using the Open FOAM package. There are six radiation models available in Open FOAM. In this work, the internal medium is diathermic. In this case, to take into account the contribution of radiative transfer to the total heat transport, it is advisable to use the view-factors method. The governing system is solved using the control volume approach. The implicit Euler scheme is employed for time discretization. For approximating the convective terms, the Gauss upwind scheme was employed. Meanwhile, to evaluate the diffusion terms, a central difference scheme was utilized. The methods used to obtain systems of linear algebraic equations were preconditioned conjugate gradient technique and preconditioned bi-conjugate gradient. The PIMPLE algorithm integrates the PISO and SIMPLE methods to manage pressure-velocity coupling in the Navier-Stokes equations. The study was conducted on a uniform grid of 120×120 elements.

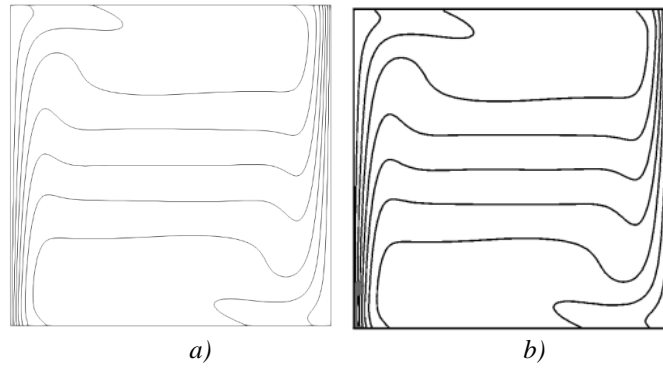
ChtMultiRegionFoam was used as the OpenFOAM solver. This is a specialized solver in the OpenFOAM library designed for modeling coupled heat transfer problems between multiple immiscible regions with different physical properties. It allows for combining calculations for solids and liquids/gases in a single formulation. The computational domain is divided into several independent subgrids. Each region has its own physical properties, equations, and boundary conditions.

The radiative Nusselt number ( $Nu_{rad}$ ) and convective Nusselt number ( $Nu_{con}$ ) are defined for each surface of the heat-generating element as follows:

$$Nu_{con} = \frac{\lambda_f \frac{\partial T}{\partial n}}{\lambda_f \frac{(T_{sf} - T_c)}{L}} \tag{7}$$

$$Nu_{rad} = \frac{q_r}{\lambda_f \frac{(T_{sf} - T_c)}{L}} \tag{8}$$

The numerical technique has been verified using the results obtained by Wang and colleagues [16] for convective-radiative heat transport within an air-filled square enclosure. Figure 2 illustrates the contrast of isotherms for surface emissivity  $\epsilon = 0.1$  and Rayleigh number  $Ra = 10^6$ .



**Figure 2.** Isotherms for  $Ra = 10^6$ ,  $\varepsilon = 0.1$ : a) present study b) Wang et al. [16].

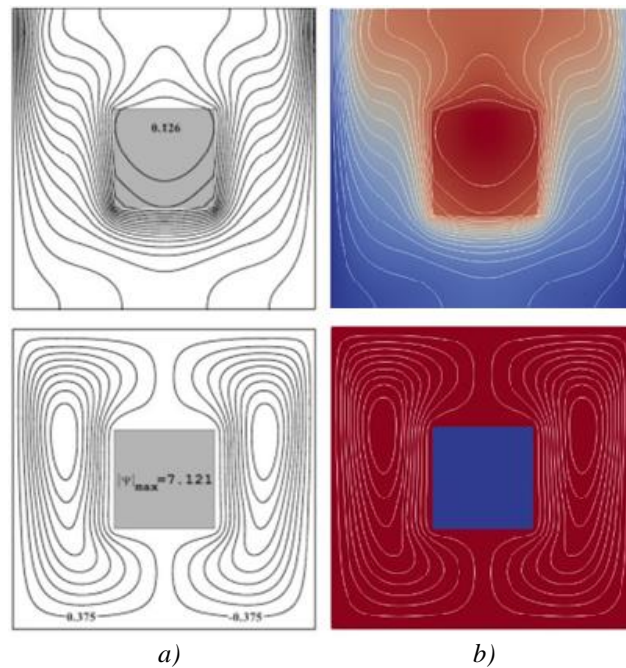
Table 2 presents the average radiative Nusselt numbers for different surface emissivities and Rayleigh numbers.

Table 2

Similarity of current estimates of radiative mean Nusselt number with published findings of Wang and colleagues [16]

$Ra$	$\varepsilon$	$Nu_{rad}$	$Nu_{rad}$ ([16])
$10^5$	0,2	1,089	1,090
$10^5$	0,8	5,188	5,196
$10^6$	0,2	2,356	2,355
$10^6$	0,8	11,271	11,265

To verify the computational approach, conjugate heat exchange in an enclosure containing a heat-producing element has been compared with the estimations made by Sivaraj et al. [17] (Figure 3).

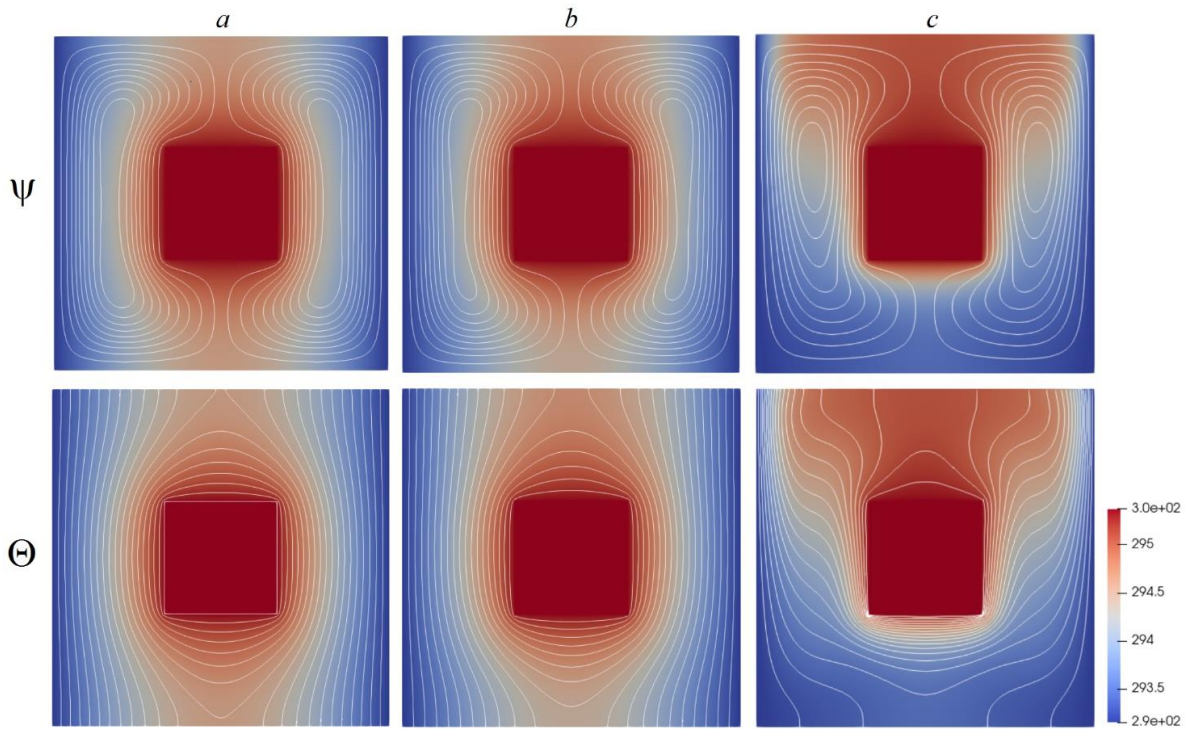


**Figure 3.** Isotherms and isolines of the stream function at  $Ra = 10^5$ ,  $\varepsilon = 0$ ,  $k_s/k_f = 10$ : a) Sivaraj et al. b) present study [17].

### Results and discussion

The examined problem was studied using the specified values for the controlling parameters: surface emissivity ( $\varepsilon = 0, 0.3, 0.6, 0.9$ ), Rayleigh number ( $Ra = 10^4, 10^5, 10^6$ ), heat-generating solid body thermal conductivity  $k_s = 149 \text{ W/(m}\cdot\text{K)}$ , the area ratio  $A = h/L$  is considered to be  $1/3$  (except for the analysis of this parameter at  $Ra = 10^6$ ), volume density of heat flux  $q_v = 719 \text{ W/m}^3$ , Prandtl number  $Pr = 0,71$ . Considerable attention has been devoted to investigating the effects of Rayleigh number and surface emissivity on fluid flow and heat transfer inside the studied cavity. The outcomes, which encompass isotherms, streamlines, and average Nusselt numbers for diverse key parameter values, are illustrated in Figures 4–5 and in Table 2.

The isotherms and streamlines obtained for various Rayleigh number values are depicted in *Figure 4*. Two symmetrical convective cells are developed inside the cavity, one of which rotates clockwise in the right half, and the other one is rotate anticlockwise in the left half, which is caused by local cooling from the vertical walls and heating from the heat-generating solid body. As the Rayleigh number increases, the centers of the convective cells move towards the top section of the cavity.



**Figure 4.** Isotherms ( $\Theta$ ) and streamlines ( $\Psi$ ) at  $\varepsilon = 0.9$ ,  $A = 1/3$ : a)  $Ra = 10^4$  b)  $Ra = 10^5$  c)  $Ra = 10^6$ .

The isotherm curvatures observed in *Figure 4c* are a result of the upward movement of fluid above the heat source region. Significant mixing of the departing cold medium with the surrounding walls occurs when a thermal plume rises from the heat source's surface. Conduction is the primary heat transmission mechanism at low Rayleigh numbers. As the buoyancy force develops in magnitude, thermal boundary layers thin out and thermal stratification forms in the cavity at  $Ra = 10^4$ .

*Table 2* displays the average radiative Nusselt number at the surface of the heat-generating solid body for varying estimates of surface emissivity and Rayleigh number. Notably, when surface emissivity is zero, the effect of radiation becomes insignificant. An increase in surface emissivity leads to a reduction in convective heat transfer because of a decrease in the temperature gradient within the cavity. Furthermore, higher surface emissivity values amplify the heat transfer via radiation, as evidenced in *Table 3*. As a result, increasing the surface emissivity from 0,6 to 0,9 at  $Ra = 10^5$  boosts the average radiative Nusselt number by a factor of 2,65.

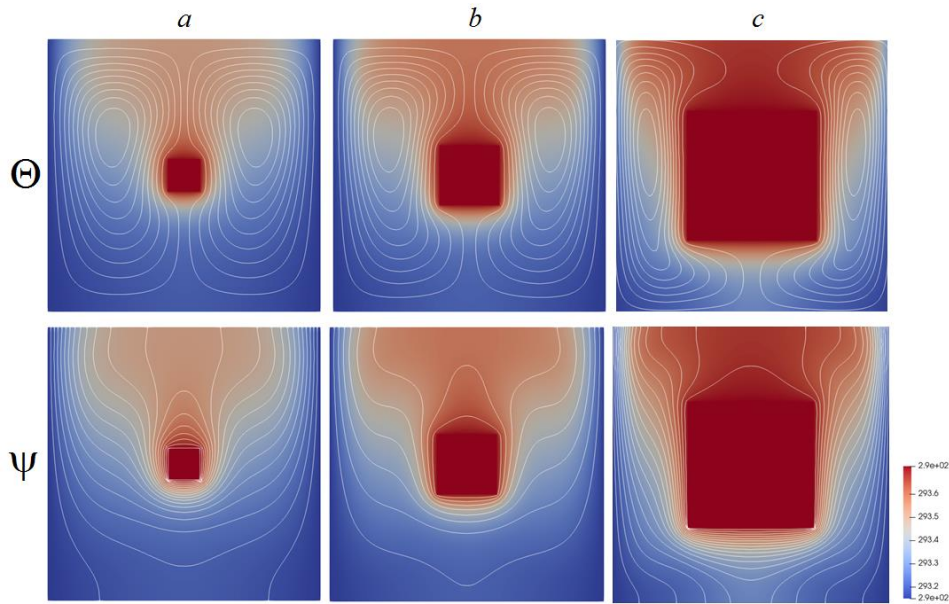
Table 3

Variations in the average radiative Nusselt number at the heat-generating surface of solid body against different estimates of Rayleigh number and emissivity of surface

$\varepsilon$	$Ra$	$Nu_{rad}$
0,3	$10^4$	1,190
	$10^5$	5,612
	$10^6$	26,031
0,6	$10^4$	2,573
	$10^5$	12,130
	$10^6$	32,133
0,9	$10^4$	4,216
	$10^5$	19,870
	$10^6$	52,823

In electronic device design, a key priority is to accurately control and foresee the temperature within the cavities. The advancement of modern electronic technologies often results in the miniaturization of heat-producing elements,

which typically leads to higher operating temperatures and subsequently impacts the consistency of the equipment. Thermogravitational convection cooling is exceptionally efficient since it doesn't depend on fans, which can be susceptible to mechanical breakdowns. Additionally, passive convective cooling is known for its reliability, cost-effectiveness, and silent operation. In this regard, we investigated the cooling of heat-generating elements of different sizes using exclusively natural convection. Isotherms and streamlines are presented in *Figure 5* for the different area ratio ( $A$ ) values.



**Figure 5.** Isotherms ( $\Theta$ ) and streamlines ( $\Psi$ ) at  $\varepsilon = 0$ ,  
 $Ra = 10^6$ : a)  $A = 1/9$  b)  $A = 2/9$  c)  $A = 4/9$ .

It is clear that for all evaluated values of  $A$ , there exist two convective cells, one rotating clockwise and the other counterclockwise. Warmed fluid ascends from the heat-generating source, contacts the upper adiabatic wall, divides, and flows toward the solid vertical walls. Consequently, these convective cells display approximately equivalent strengths. Notably, the horizontal dimensions of these cells diminish as the size of the heater expands. The distribution of isotherms reflects the formation of a thermal plume over a source of volumetric heat release. Analysis of the integral characteristics of heat exchange shows that when changing the parameter  $A$  from  $1/9$  to  $4/9$ , the mean heat transfer coefficient changed by 15%.

### Conclusion

This research focuses on the dynamics of free convection and surface radiation in an air-filled cavity holding a heat-generating body. Given the widespread use of comparable designs in the electronics sector, this study is highly relevant from an engineering standpoint. The finite volume approach has been utilized to solve the governing system, which encompass a wide range of dimensionless parameters. The consequences of Rayleigh number and surface radiation on fluid flow and thermal transmission have been thoroughly studied for the variety of parameter combinations. Notably, it has been shown that when the surface emissivity increased from 0,3 to 0,9, the mean radiative Nusselt number increased by up to 3,54 times. The results show that surface radiation has a significant effect on heat transport, especially when the estimate of emissivity is high. These findings imply that heat removal may be efficiently increased by even a slight increase in surface emissivity in a sealed electronic enclosure with a single heat-producing solid body. In the future, it is planned to implement a numerical model for the analysis of three-dimensional effects.

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